Equations of Motion of the Asteroid Moving in Earth's Atmosphere

$$\frac{d\vec{v}}{dt}(\vec{r}) \;=\; \vec{g} \;-\; \frac{c_d}{2}\,\frac{\rho_{at}(x)\vec{v}(\vec{r})^2\,\pi\,R(t)^2}{m}\,\vec{e}_t$$

Equation of Deformation of the Ellipsoid Shaped Asteroid

$$\frac{dv_q}{dt} \;=\; \frac{\rho_{at}(x)}{\rho_{imp}}\,\frac{\vec{v}(\vec{r})^2}{R(t)}$$

Density of the Exponential Atmosphere (isotherm)

$$\rho_{at}(x) \;=\; \rho_0\,\exp\left(-\frac{m_p\,g\,x}{k_b\,T_{amb}}\right)$$

Conservation Equations for a Gas Dynamic Flow Field

$$\frac{\partial}{\partial t}\,\rho + \vec{\nabla}\cdot\,\rho\,\vec{v} \;=\; 0$$

$$\frac{\partial}{\partial t}\,\rho\,\vec{v} + \vec{\nabla}\cdot\,\rho\,\vec{v}\otimes\vec{v} \;=\; -\vec{\nabla}p \,+\rho\,\vec{g}$$

$$\frac{\partial}{\partial t}\,e + \vec{\nabla}\cdot\,(e\,+p)\;\vec{v} \;=\; \rho\,\vec{g}\cdot\vec{v}$$

$$p \;=\; (\kappa-1)\left(e-\frac{1}{2}\rho\vec{v}^2\right)$$

Conservation Equations for the Hypervelocity Impact

$$\frac{\partial}{\partial t}\rho_i + \vec{\nabla}\cdot\ \rho_i\,\vec{v} \;=\; 0,\; i \in \{1,2,3\}$$

$$\rho \;=\; \rho_1 \;+ \rho_2 \;+ \rho_3$$

$$p \;=\; p_1 \;+ p_2 \;+ p_3$$

$$\frac{\partial}{\partial t}\rho\,\vec{v} + \vec{\nabla}\cdot\ \rho\,\vec{v}\otimes\vec{v} \;=\; -\vec{\nabla}p + \rho\,\vec{g}$$

$$\frac{\partial}{\partial t}e + \vec{\nabla}\cdot\ (e\;+p)\;\vec{v} \;=\; \rho\,\vec{g}\cdot\vec{v}$$

$$\varepsilon_i \;=\; \frac{\rho_i}{\rho}e \;-\; e_i,\; i \in \{1,2\}$$

$$\frac{\partial}{\partial t}\varepsilon_i + \vec{\nabla}\cdot\ \varepsilon_i\,\vec{v} \;=\; \left(p_i \;-\; \frac{\rho_i}{\rho}p\right)\vec{\nabla}\cdot\vec{v},\; i \in \{1,2\}$$

$$e_3 \;=\; e \;-\; (e_1 \;+\; e_2)$$

Governing Equations for an Incompressible Flow Field for Large Density Contrasts

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}\cdot\ \rho\,\vec{v} \;=\; 0$$

$$\frac{\partial}{\partial t}\rho + \left(\vec{\nabla}\rho\right)\ \cdot\vec{v} \;=\; 0$$

$$\vec{\nabla}\cdot\vec{v} \;=\; 0$$

$$\frac{\partial}{\partial t}\vec{v} + \vec{v}\cdot\vec{\nabla}\,\vec{v} \;=\; -\frac{1}{\rho}\vec{\nabla}p \;+ \vec{g}$$

$$\vec{\nabla}\cdot\left(\frac{1}{\rho}\vec{\nabla}\,p + \vec{v}\cdot\vec{\nabla}\,\vec{v}\right) \;=\; 0$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\Omega - \vec{v}\right) \;=\; 0$$

$$p \;=\; p + \frac{d\Omega}{dt}$$

$$\vec{v} \;=\; \vec{v} \;-\; \vec{\nabla}\Omega$$

Shallow Water Equations

$$\frac{\partial h}{\partial t} + \vec{\nabla} \cdot (h\,\vec{v}) \;=\; 0$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla}\vec{v} + g\vec{\nabla}\;(h+s) = \;0$$

One Dimensional Shallow Water Equations on a Sphere

$$\frac{\partial h}{\partial t} \;+\; \frac{1}{R_E}\left(\frac{\partial}{\partial \theta}\,(h\,v) \;+\; \cot(\theta)\,h\,v\right) \;=\; 0$$

$$\frac{\partial v}{\partial t} \;+\; \frac{1}{R_E}\left(v\,\frac{\partial v}{\partial \theta} \;+\; g\,\frac{\partial h}{\partial \theta}\right) \;=\; 0$$

Displacement Energy and Volume of Parabolic Water Cavity

$$E_{cv} \;=\; \frac{\pi}{6}\rho_w\,g\,R^2\,t^2$$

$$V_{cv} \;=\; -\frac{\pi}{2}\,t\,R^2$$

Energy and Volume of Breakwater in Worse Case Estimation

$$E_{ts} \;=\; \frac{\pi}{3}\,\rho_w\,g\,R_E\,\sin\left(\frac{b}{R_E}\right)\left(5\,h^3 \;+\; d^3\,\tan^2(\alpha)\right)$$

$$V_{ts} \;=\; \pi\,R_E\,\sin\left(\frac{b}{R_E}\right)\left(2\,h^2 \;-\; d^2\,\tan(\alpha)\right)$$

Nomenclature

θ	latitude	$\vec{e}_t$	tangential unit vector
E	energy	e	energy density
$\vec{g}$	gravity acceleration	h	water height, breakwater height
k_b	Boltzmann constant	κ	adiabatic coefficient
p	pressure	R	radius
$\vec{r}$	spatial vector	ρ	density
$\vec{v}$	velocity	x	atmospheric height
d	low tide distance	s	ocean bottom topography
m	mass	T	temperature
c	coefficient	Ω	gauge potential
t	depht	V	volume
b	distance	$\vec{r}$	spatial vector

Subscripts

amb	ambient	at	atmospheric
d	drag	E	earth
imp	impactor	p	atmospheric particle
ts	tsunami	0	ground level
w	water	t	tangent
cv	cavity	q	ellipsoid boundary